To wrap up our fall semester:

An Open problem:

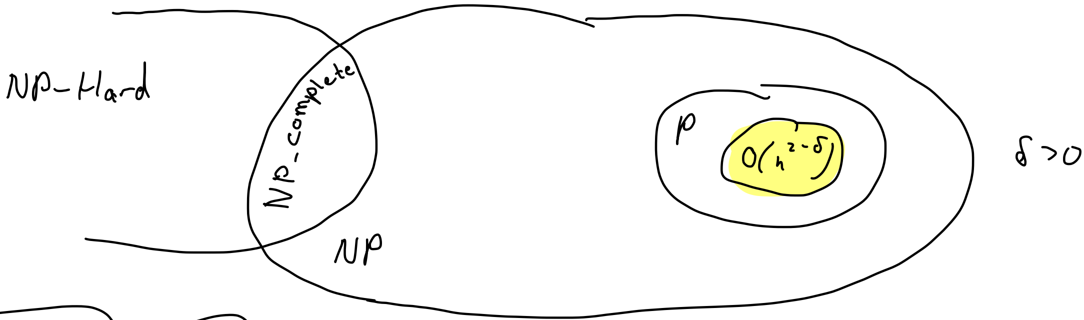
Is computing Fréchet distance 3SUM-Hard?

Recall (from last week), There is no $O(n^{2-\delta})$ algorithm for the (cont's or discrete) Fréchet distance for any $\delta > 0$ unless SETH' fails.

SETH' → There is no $O^*((2-\delta)^N)$ algo for CNF-SAT for any $\delta > 0$. (O^* hides poly. factors)

Only known lower bound for Fréchet is $\Omega(n \log n)$. It is completely open if Fréchet is 3SUM-Hard, with light controversy. (Details to come)

Brief Perspective:



Recall **3SUM**: (Brad's Talk from end of summer)

Input: A set S of n integers

Output: Yes or No, does there exist $a, b, c \in S$ such that $a + b + c = 0$? $a \neq b$ $b \neq c$ $a \neq c$

Eg) $S = \{57, -42, -11, 15, 6, 37, 20, 5\}$

Solution: $37 - 42 + 5 = 0$

Eg (cont'd)

$$S = \{57, -42, -11, 15, 6, 37, 20, 5\}$$

Algorithmic Approach?

- Naive: Check all subsets of size 3 $(O(n^3))$
- $O(n^2)$ algo w/ a little more cleverness.

Sort S & make 2 copies:

$(-42, -11, 5, 6, 15, 20, 37, 57)$

p_1 p_3

$(-42, -11, 5, 6, 15, 20, 37, 57)$

p_2

1. $(-42 + 57) - 11 = 4 > 0$, decrease p_2
2. $(-42 + 37) - 11 = -16 < 0$, no sum, increase p_3
3. $(-42 + 37) + 5 = 0$ ✓

$O(n^2) \rightarrow$ Worst case, touches all other elements for each $s \in S$.

Hypothesis: 3SUM is unsolvable in $O(n^{2-\delta})$ time $\delta > 0$.

3SUM Hardness:

A problem (like the Fréchet distance???) is 3SUM-Hard if solving it in subquadratic time implies $\exists$ subquadratic solution to 3SUM.

Instance of known hard problem (3Sum) $\rightarrow$ Instance of studied problem (Fréchet) $\rightarrow$ Sol'n to Fréchet $\rightarrow$ sol'n to 3SUM

Example of a slick 3SUM reduction: (Thanks, Brad)

Gajentaan - Overmars (1995)

GeomBase Problem:

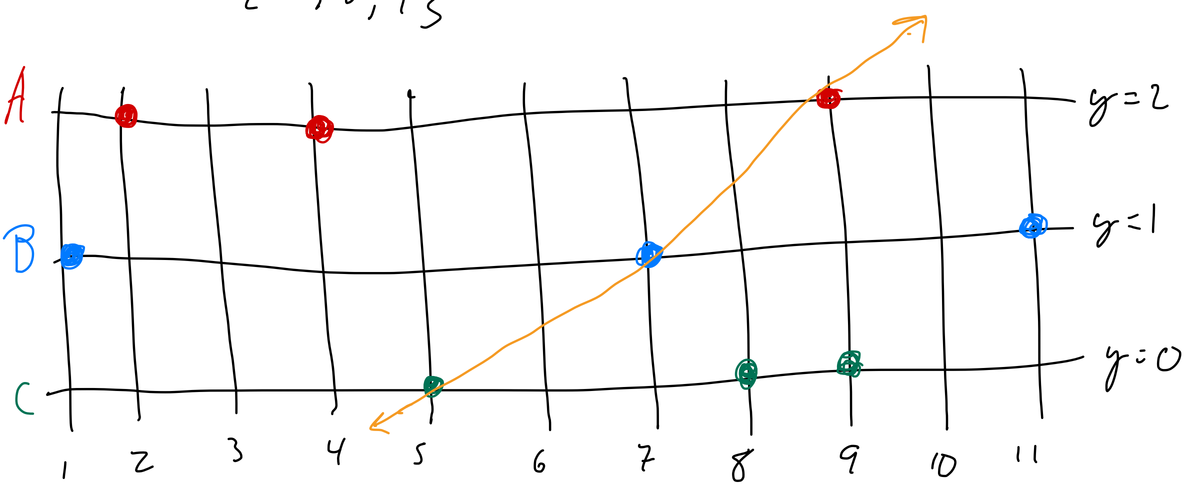
<u>Input</u> n points on the lines $y=0$, $y=1$, $y=2$,

<u>Output</u> Yes/No, is there a non-horizontal line that goes thru 3 points?

Eg) $A = \{2, 4, 9\}$

$B = \{1, 7, 11\}$

$C = \{5, 8, 9\}$



Reduction: Given $S \subset \mathbb{Z}$, $|S| = n$,

there exists $a \neq b \neq c \in S$ s.t. $a+b+c=0$

if and only if A, $-\frac{B}{2}$, C partitions of S of length $n/3$ solve Geom Base.

pf: To solve Geom Base, need

$b - a = c - b$ for $a \in A$, $b \in B$, $c \in C$.

Then

$$2b = c + a,$$

and $0 = c + a - 2b$.

So negate + half all $b \in B$, + otherwise leave A, C alone.

Geom Base is 3SUM-Hard.