2.5 Schmidt decomp. + purifications

'Spose $|\psi\rangle$ is a pure state of a composite system, AB. Then $\exists$ orthonormal $|i_A\rangle$ for system A, $|i_B\rangle$ for system B, s.t.

$$|\psi\rangle = \sum_i \lambda_i |i_A\rangle|i_B\rangle \quad \otimes$$

where $\lambda_i \in \mathbb{R}^+$ with $\sum_i \lambda_i^2 = 1$

Consequence:

Let $|\psi\rangle$ as before. Then $\rho^A = \sum_i \lambda_i^2 |i_A\rangle\langle i_A|$ and $\rho^B = \sum_i \lambda_i^2 |i_B\rangle\langle i_B|$.

So eigenvalues of ρ^A and ρ^B are λ_i^2.

eg: $(|00\rangle + |01\rangle + |11\rangle)/\sqrt{3}$

$$\mathrm{tr}((\rho^A)^2) = \mathrm{tr}((\rho^B)^2) = 7/9$$

If we have a pure state of a composite system, important properties determined by eigen. of ρ have to be the same.

2.6] EPR and the Bell inequality:

Non-classical vs. classical: what's the difference between worlds?
Bell ineq. a good example!

For "normal" objects existence is independent of observation.
measurements reveal physical properties.

Quantum view is different, existence doesn't seem independent of observation

Notable Objectors: Einstein!

Represent reality completely in theory

Anti-Correlation in the $\boxed{\text{EPR}}$ experiment:

'Spose we have the 2 qubit state

$$|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$ → entangled state of 2 qubit system

If we measure the spin on $\vec{v}$ axis for both qubits, (eq'n 2.116 on pg. 90)
getting $\boxed{+1 \text{ or } -1}$ for each qubit,

- Results of the two measurements are always opposite one another. !!

To see why: 'spose $|a\rangle$ and $|b\rangle$ eigenstates of $\vec{v}\cdot\vec{\sigma}$. Then $\exists\, \alpha, \beta, \gamma, \delta$
s.t. $|0\rangle = \alpha|a\rangle + \beta|b\rangle$, $|1\rangle = \gamma|a\rangle + \delta|b\rangle$.

spin on $\vec{v}$ axis

Substituting Gives: $$\frac{|01\rangle - |10\rangle}{\sqrt{2}} = (\alpha\delta - \beta\gamma)\frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$$

But $\alpha\delta - \beta\gamma$ is just $\det\left(\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}\right) = e^{i\theta}$ for $\theta \in \mathbb{R}$.

So $\frac{|01\rangle - |10\rangle}{\sqrt{2}} = \frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$

Let entangled $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$ belong to Alice + Bob.

Suppose Alice measures $+1$. (Bob measures -1 spin $\vec{v}$

For EPR, physical properties must correspond to an element of reality. However, standard quantum mechanics only tells one how to calculate such probabilities if $\vec{v} \cdot \vec{\sigma}$ is measured.

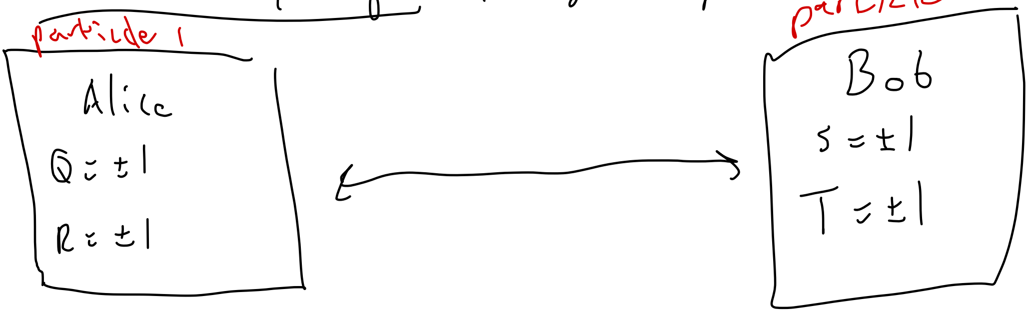
No fundamental element representing $\vec{v} \cdot \vec{\sigma}$ for all unit $\vec{v}$.
spin

For EPR, quantum framework incomplete.

But nature has had the last laugh.

want to return to classical framework

Bell's Inequality: (thought experiment)



Charlie prepares 2 particles, and sends one to Alice, one to Bob. Say Alice has two properties she could measure, P_Q and P_R. She decides randomly which to measure when she recieves the particle. Bob does the same with P_S and P_T.

Algebraic Reasoning: $QS + RS + RT - QT$

$$QS + RS + RT - QT = (Q+R)S + (R-Q)S$$

$\Rightarrow$ either (i) $(Q+R)S = 0$ or (ii) $(R-Q)T = 0$

$\Rightarrow$ $QS + RS + RT - QT = 2$

Now 'spose $p(q,r,s,t)$ is probability $Q=q$, $R=r$, $S=s$, $T=t$.
Let $E(\cdot)$ denote mean of a quantity:

①
$$E(QS + RS + RT - QT) = \sum_{qrst} p(q,r,s,t)(qs + rs + rt - qt)$$
$$\leq \sum_{qrst} p(q,r,s,t) \times 2$$
$$= 2$$

And also:

(2) $E(QS+RS+RT-QT) = \sum_{qrst} p(q,r,s,t)\,qs + \sum_{qrst} p(q,r,s,t)\,rs + \sum_{qrst} p(q,r,s,t)\,rt - \sum_{qrst} p(q,r,s,t)\,qt$

$= E(QS) + E(RS) + E(RT) - E(QT)$

Comparing both equations 1 & 2 gives the Bell inequality:

$$E(QS) + E(RS) + E(RT) - E(QT) \leq 2.$$

Back 2 Quantum:

Let Charlie prepare $|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$, and give one qubit to Bob & Alice.

They observe $Q = Z_1$, $S = \frac{-Z_2 - X_2}{\sqrt{2}}$

$R = X_1$, $T = \frac{Z_2 - X_2}{\sqrt{2}}$

Then average values are:

$\langle QS\rangle = \frac{1}{\sqrt{2}}$, $\langle RS\rangle = \frac{1}{\sqrt{2}}$, $\langle RT\rangle = \frac{1}{\sqrt{2}}$, $QT = -\frac{1}{\sqrt{2}}$

so

$\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = \boxed{2\sqrt{2}}$!! > 2

Is this just a fluke?

No! Nature verifies this experimentally.

So the proof of Bell ineq. must have faulty logic somewhere.

2 sketchy ?? assumptions are made:

(1) Realism. P_Q, P_R, P_S, P_T have definite values Q, R, S, T that exist independent of observation.

(2) Locality. Alice's measurement doesn't affect Bob's measurement,

The world ain't locally realistic!

What would some entanglement buy me in this problem?